Photoproduction of the Hypertriton

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In the framework of the impulse approximation we study the photoproduction of the hypertriton ${}^3_\Lambda$H by using realistic ^{3}He wave functions obtained as solutions of Faddeev equations with the Reid soft-core potential for different ${}^3_\Lambda$H wave functions. We obtain relatively small cross sections of the order of 1 nb. We also find that the influence of Fermi motion is important, while the effect of different off-shell assumptions on the cross section is not too significant.

1. INTRODUCTION

Photoproduction of the hypertriton can provide new information on the YN interaction, which up to now is only poorly known from the available YN scattering data. Being the lightest hypernucleus, the hypertriton is obviously the first system in which the YN potential, including the interesting Λ-Σ conversion potential, can be tested in the nuclear environment. This is also supported by the fact that neither the ΛN nor the ΣN interactions are sufficiently strong to produce a bound two-body system. Therefore the hypertriton will play an important role in hypernuclear physics, similar to the deuteron in nuclear physics. Although hypernuclear systems have been extensively studied for a wide range of nuclei by means of hadronic processes such as stopped and low momentum kaon induced reactions, $\mathrm{A}(K,\pi)_\Lambda\mathrm{B}$, as well as $\mathrm{A}(\pi,K)_\Lambda\mathrm{B}$ reactions, electromagnetic productions will, at some point, be required for a complete understanding of hypernuclear spectra.

In this work we investigate the reaction ${}^3\mathrm{He}(\gamma,K^+){}^3_\Lambda\mathrm{H}$, i.e. the incoming real photon interacts with a nucleon (proton) in ^{3}He creating a lambda which combines with the other two nucleons to form the bound hypertriton and a positively charged kaon which exits the nucleus. To our knowledge, no analysis has been made and no experimental data are available for this reaction. The only related work is due to Komarov *et al.* [1], who investigated the complementary reaction, $p + d \rightarrow K^+ + {}^3_\Lambda\mathrm{H}$, and estimated that at an incident proton energy $T_p = 1.13$ - 3.0 GeV, the maximum differential cross section is well below 1 nb/sr.

2. THE THREE-BODY WAVE FUNCTIONS

In our formalism, the ^{3}He wave functions are expanded in orbital momentum, spin, and isospin of the pair (2,3) and the spectator (1) with the notation

$$\Psi_{^3\mathrm{He}}(\vec{p},\vec{q}) \;=\; \sum_\alpha \phi_\alpha(p,q)\; |(Ll)\mathcal{L},(S\tfrac{1}{2})\mathcal{S},\tfrac{1}{2}M\rangle\; |(T\tfrac{1}{2})\tfrac{1}{2}M_t\rangle\;, \tag{1}$$

where $\vec{p}$ ($\vec{q}$) denotes the momentum of the pair (spectator) and $\phi_\alpha(p,q)$ stands for numerical solutions of Faddeev equations using the realistic nucleon-nucleon potential [2]. In Eq. (1) we have introduced $\alpha = \{Ll\mathcal{L}S\mathcal{S}T\}$ to shorten the notation, where L, S, and T are the total angular momentum, spin, and isospin of the pair (2,3), while for the spectator (1) the corresponding observables are labelled by l, $\frac{1}{2}$, and $\frac{1}{2}$, respectively.

For the hypertriton we choose the simple model developed in Ref. [3], which should be reliable enough to obtain a first estimate for the photoproduction of the hypertriton. Using the same notation as in Eq. (1) the wave function may be written as

$$\Psi_{^3_\Lambda\mathrm{H}}(\vec{p},\vec{q}) \;=\; \sum_\alpha \phi_\alpha(p,q)\; |(Ll)\mathcal{L},(S\tfrac{1}{2})\mathcal{S},\tfrac{1}{2}M\rangle\;, \tag{2}$$

where $\phi_\alpha(p,q)$ is given by the two separable wave functions of the deuteron and the lambda, $\phi_\alpha(p,q) \;=\; \Psi_d^{(L)}(p)\;\varphi_\Lambda(q)$, with the lambda part of the wave functions obtained by solving the Schrödinger equation for a particle moving in the Λ-d effective potential.

While using this simple wave function for most calculations, we also compared with the results for the correlated Faddeev wave function of Ref. [4] in order to probe the sensitivity of the cross section to different descriptions of the hypertriton. In spite of a more complicated structure, the wave function may still be written in the form of Eq. (1).

3. THE CROSS SECTIONS

In the lab system the cross section for kaon photoproduction off ^{3}He is

$$\frac{d\sigma_\mathrm{T}}{d\Omega_K} \;=\; \frac{|\vec{q}_K^{\,\mathrm{c.m.}}|}{|\vec{k}^{\mathrm{c.m.}}|}\,\frac{M_{^3\mathrm{He}}E_{^3_\Lambda\mathrm{H}}}{64\pi^2W^2}\,\sum_\epsilon\sum_{M,M'}\;|T_\mathrm{fi}|^2\;, \tag{3}$$

where $\vec{q}_K^{\,\mathrm{c.m.}}$, $\vec{k}^{\mathrm{c.m.}}$, and W represent the momentum of kaon, photon, and the total energy in the c.m. system, respectively.

Since both initial and final states of the nucleus are unpolarized, the sums over the spin projections can be performed by means of $\sum_{M,M'}|T_\mathrm{fi}|^2 = \sum_{\Lambda,m_\Lambda}|T^{(\Lambda)}_{m_\Lambda}|^2$, with

$$\begin{aligned}T^{(\Lambda)}_{m_\Lambda} \;=\;& \sqrt{\frac{2}{\pi}}\left(\frac{E_{^3\mathrm{He}}E_{^3_\Lambda\mathrm{H}}}{M_{^3\mathrm{He}}M_{^3_\Lambda\mathrm{H}}}\right)^{\frac{1}{2}}\times\\ & \sum_{\alpha,\alpha',n}\Bigg[\,i^n\;\hat{n}\hat{\mathcal{L}}\hat{S}'\hat{S}(-1)^{n+\mathcal{S}-\frac{1}{2}}\left\{\begin{array}{ccc}\mathcal{S}' & \mathcal{S} & n\\ \frac{1}{2} & \frac{1}{2} & 1\end{array}\right\}\left\{\begin{array}{ccc}\mathcal{L} & \mathcal{S} & \frac{1}{2}\\ L & \mathcal{S}' & \frac{1}{2}\\ l & n & \Lambda\end{array}\right\}\delta_{LL'}\delta_{S1}\delta_{T0}\;\times\\ & \int d^3\vec{q}\;p^2dp\,\left(\frac{m_\mathrm{i}m_\mathrm{f}}{E_\mathrm{i}E_\mathrm{f}}\right)^{\frac{1}{2}}\varphi_\Lambda(q')\;\Psi_d^{(L)}(p)\;\phi_\alpha(p,q)\;\Big[\mathbf{Y}^{(l)}(\hat{\vec{q}})\otimes\mathbf{K}^{(n)}\Big]^{(\Lambda)}_{m_\Lambda}\Bigg],\end{aligned} \tag{4}$$

where $\mathbf{K}^{(0)} = L$ and $\mathbf{K}^{(1)} = \vec{K}$, i.e. the spin-independent and spin-dependent elementary amplitudes [5]. Since the tensor $\mathbf{K}^{(n)}$ contains complicated functions of the integration variables $\vec{q}$ and $\hat{\vec{q}} = \Omega_q$, the integral in Eq. (4) has to be performed numerically.

The tensor operators, $[\mathbf{Y}^{(l)}(\hat{\vec{q}}) \otimes \mathbf{K}^{(n)}]^{(\Lambda)}_{m_\Lambda}$, which determine the specific nuclear transitions in the reaction, are given in Ref. [5]. In contrast to the case of pion production off ^{3}He [6], the tensor operators in our case are simplified by the approximation that the hypertriton wave function only contains the partial wave with $l' = 0$. However, for future studies involving all partial waves of the advanced hypertriton model [4], the complete operator will be needed. For this purpose, we have also derived the form of Eq. (4) for the more general case [7].

4. RESULTS AND DISCUSSION

As a check of our calculations and computer codes, we compare the full result with two simple approximations. First, we reduce the cross section by allowing only S–waves to contribute to the amplitudes in Eq. (4). In this approximation we obtain

$$\frac{d\sigma(^3\mathrm{He})}{d\sigma(p)} \approx \frac{|\tilde{\vec{K}}|^2}{|\vec{K}|^2} \approx 1.8 \times 10^{-3} , \tag{5}$$

where the tilde denotes the integration over the internal momenta $\vec{p}$ and $\vec{q}$ weighted by the two wave functions. At $k = 1.8$ GeV the elementary reaction model of Ref. [8] yields a maximum cross section of about 500 nb/sr. Hence, the corresponding cross section on ^{3}He will be about 1 nb/sr at most.

As a second approximation, we consider the struck nucleon inside ^{3}He as having a fixed momentum. Therefore, the L and $\vec{K}$ amplitudes can be factored out of the integral and the cross section off ^{3}He may be written in terms of the nuclear form factor $F(Q)$ and the elementary differential cross section,

$$\frac{d\sigma_{\mathrm{T}}}{d\Omega_K} = \tfrac{1}{9}\, W_A^2\, |F(Q)|^2 \left(\frac{d\sigma_{\mathrm{T}}}{d\Omega_K}\right)_{\mathrm{proton}} , \tag{6}$$

where W_A represents the kinematical factor given, i.e., in Ref. [5]. The result is displayed in Fig. 1. The nuclear cross section at forward angles is smaller than in the case of elementary kaon production by two orders of magnitude. As $\theta_K^{\mathrm{c.m.}}$ increases, the cross section drops quickly, because the momentum transfer to the nucleus increases as function of $\theta_K^{\mathrm{c.m.}}$. Therefore, the cross section is very small. The underlying reason is the lack of high momentum components in the $^3_\Lambda$H wave function. Since the momentum transfers are high, the lambda momentum is high as well, which inhibits hypernuclear formation.

Figure 1 also shows the significant difference between the cross sections calculated with the approximation of Eq. (6) and the full result obtained from Eq. (3). This discrepancy is due to the *factorization* approximation. It can be seen that in the full calculation the integrations of both spin-independent and spin-dependent amplitudes over the internal momenta lead to destructive interferences and a further reduction of the cross section.

In contrast to our previous conjecture, Fig. 2 shows the significance of the higher partial waves which reduce the cross section by a factor of more than three. The reason can be

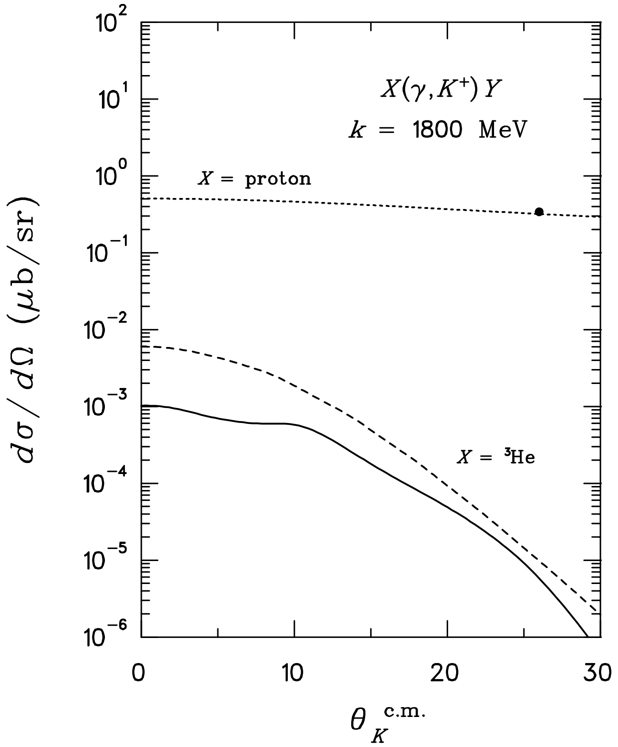

Figure 1. Differential cross section for kaon photoproduction off the proton and ^{3}He as function of kaon angle. The elementary reaction (dotted line) is taken from Ref. [8] and the corresponding experimental datum is from Ref. [9]. The dashed line shows the approximation for production off ^{3}He calculated from Eq. (6), the solid line represents the exact calculation using S-waves.

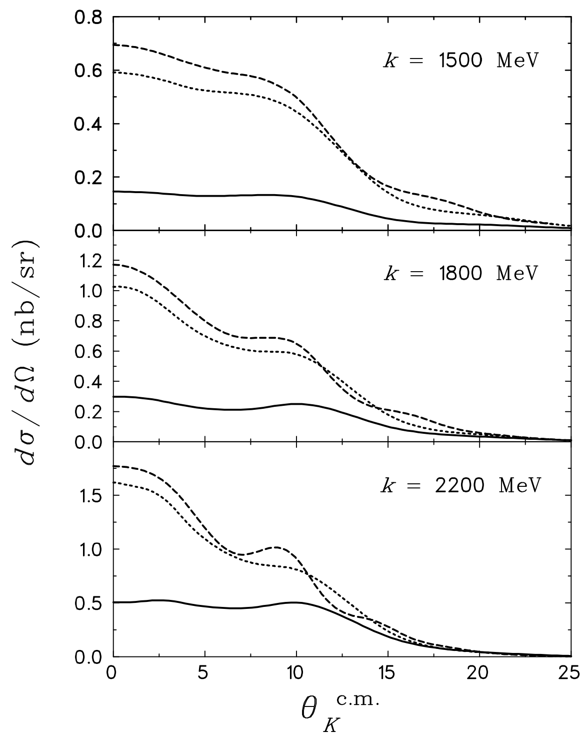

Figure 2. The cross section for kaon photoproduction off ^{3}He at three different excitation energies. The dotted curves are obtained from the the calculation with S–waves only and the simple hypertriton wave function, the dashed curves are obtained with S–waves only and the correlated Faddeev wave function of Ref. [4], while the solid curves show the result after using all of the partial waves and the simple hypertriton wave function.

traced back to Eq. (4). In spite of its small amplitude, the transition from $\alpha = 8$ to $\alpha' = 1$ may not be neglected, since $\alpha' = 1$ is the most likely state in the hypertriton. We also note that the angular momentum part of the tensor amplitude in Eq. (4) yields a considerably large contribution for this transition. In comparison, the higher partial waves in pion photo- and electroproduction decrease the cross section by at most 15% and 20%, respectively.

Since the (γ, K) process is a high momentum transfer process and the simple analytical hypertriton wave function used until now contains no short-range correlations, we also show in Fig. 2 a comparison with the correlated three-body wave function of Ref. [4] that includes a proper short-range behavior. While the cross section obtained with the Faddeev wave function shows more structures, the differences are only of order 10-20%.

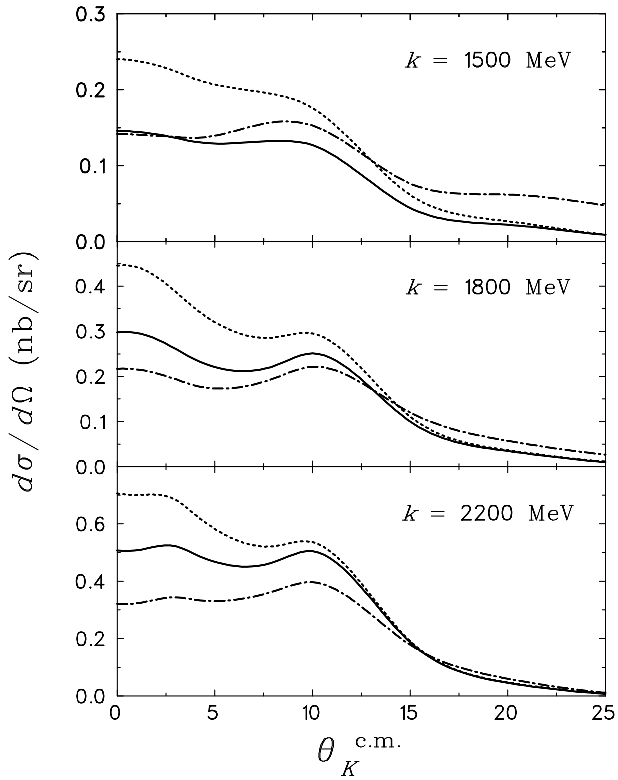

Figure 3. The influence of Fermi motion on the differential cross section at three different photon energies. The dash-dotted (dotted) curve is the *frozen nucleon* (average momentum) approximation, the solid curve shows the exact result.

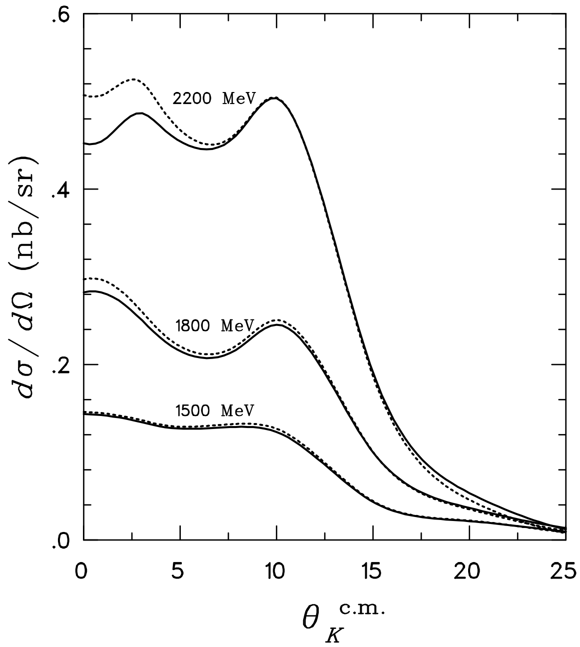

Figure 4. The effect of different off-shell assumptions on the cross section calculated at three different energies. The dotted curves have been calculated with the initial nucleon on-shell, the solid curves with the final hyperon on-shell.

The absence of short-range correlations in the simple hypertriton model does only become obvious for much larger momentum transfers.

The small size of the cross section obtained raises the question of a possible significance of two-step processes, such as $\gamma + p \rightarrow p + \pi^0 \rightarrow K^+\Lambda$. Two-step processes were studied in Ref. [10] for pion photoproduction on ^{3}He and found to be significant only at Q^2 much larger than in our case. Ref. [11] also included these processes in η photoproduction on the deuteron and found small effects.

We have investigated the contribution of non-localities generated by Fermi motion in the initial and final nuclei. As in former studies [6], an exact treatment of Fermi motion is included in the integrations over the wave functions in Eq. (4), while a local approximation can be carried out by freezing the operator at an average nucleon momentum $\langle \vec{k}_1 \rangle = -\kappa(A-1)\vec{Q}/2A$, where $A = 3$ in this case. A value of $\kappa = 0$ corresponds to the *frozen nucleon* approximation, whereas $\kappa = 1$ yields the average momentum approximation. The latter case has been shown to yield satisfactory results for pion photoproduction in the s- and p-shells [12]. Figure 3 compares the cross sections calculated in the two ap-

proximations with the exact calculation. There appears a systematic discrepancy between the calculation with Fermi motion and the one with the average momentum approximation at all energies. Unlike in pion photoproduction, the average momentum approximation cannot simulate Fermi motion in kaon photoproduction, and the discrepancies between the different methods, especially near forward angles, are too significant to be neglected.

Finally, we show the effect of different off-shell assumptions on the cross section in Fig. 4. The nucleons in the initial and final states are clearly off-shell. However, the elementary amplitudes are constructed for on-shell nucleons in the initial and final states. For this reason, we test the prescriptions given in Ref. [6], i.e. we assume that (1) the initial nucleon is on-shell and the final hyperon off-shell, and (2) the final hyperon is on-shell and the initial nucleon off-shell. Both assumptions are compared in Fig. 4, where we see that the difference is not too significant. The largest discrepancy of 10% occurs at $k = 2200$ MeV in the forward direction as was also observed in the case of pion photoproduction.

In conclusion, we find that the hypertriton photoproduction could provide a sensitive test of the hypertriton wave functions. The small cross sections and the required high resolution measurements would become a new challenge to experimentalists.

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